Dot products

in $\mathbb{R}^2$, $\mathbb{R}^3$, or any dimensions

Multiply a vector with a vector to get a scalar.
(It is also called the scalar product)

Ex $\langle 1,2,-3\rangle \cdot \langle -4,5,6\rangle$

$$= 1(-4) + 2(5) + (-3)(6)$$

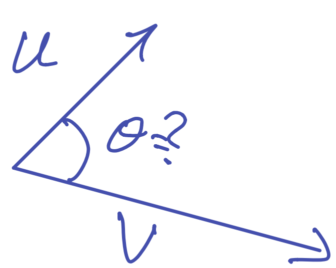
$$= -12$$

Rules for dot products:
Most obvious rules are true:

$$a \cdot b = b \cdot a$$

$$(a+b) \cdot c = (a \cdot c) + (b \cdot c)$$

Angles between two vectors in $\mathbb{R}^3$



$u = \langle u_1, u_2, u_3 \rangle$
$v = \langle v_1, v_2, v_3 \rangle$

$$\theta = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|u||v|}\right)$$

$$= \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$$

Ex1 Find the angle between
$v = i - 2j - 2k$
& $u = 6i + 3j + 2k$
in degrees.

$v \cdot u = (1)6 + (-2)3 + (-2)(2) = -4$

$|u| = 7$
$|v| = 3$

$$\theta = \cos^{-1}\left(\frac{-4}{3 \cdot 7}\right) \approx 100.98^\circ$$

orthogonal vectors
when two vectors are $\perp$.

Vector u & v are

orthogonal if $u \cdot v = 0$.

The vetor projection of u & v is the vector

$$\text{proj}_v(u) = \left(\frac{u \cdot v}{|v|^2}\right) v \qquad \frac{u}{|v|} \cdot \frac{v}{|v|}$$

The scalar component of u in the direction of v is the scalar

$$|u| \cos\theta = \frac{u \cdot v}{|v|} = u \cdot \frac{v}{|v|}$$

unit vector of v.

Ex) Find the vector projection of $u = 6i + 3j + 2k$ onto $v = i - 2j - 2k$ & the scalar component of u in the direction of v.

Ans: we find $\text{proj}_v(u)$

$$\left(\frac{u \cdot v}{|v|^2}\right) v = \frac{-4}{9}(i - 2j - 2k)$$

$$= -\frac{4}{9}i + \frac{8}{9}j + \frac{8}{9}k$$

lets find the scalar component of u in the direction of V

$$|u| \cos\theta = u \cdot \frac{V}{|V|}$$

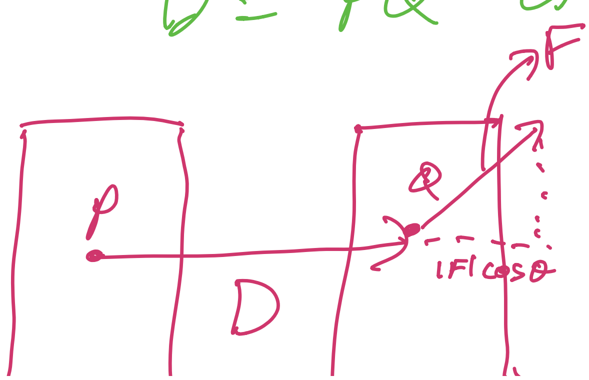
$$= (6i + 3j + 2k) \cdot \left(\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}k\right)$$

$$= 2 - 2 - \frac{4}{3} = -\frac{4}{3}$$

Application:

The work done by a constant Force F acting through a displacement

$D = \overrightarrow{PQ}$ is $W = F \cdot D$



The work done by a constant force F during a displacement D is

$|F| \cos\theta \, |D|$ which is a dot product of $F \cdot D$

Ex) If $|F| = 40$ N (newtons),

$|D| = 3$m & $\theta = 60^\circ$

the work done by F is acting from P to Q

$$W = F \cdot D$$
$$= |F|\,|D| \cos\theta$$
$$= 60 \text{ J (joules)}$$